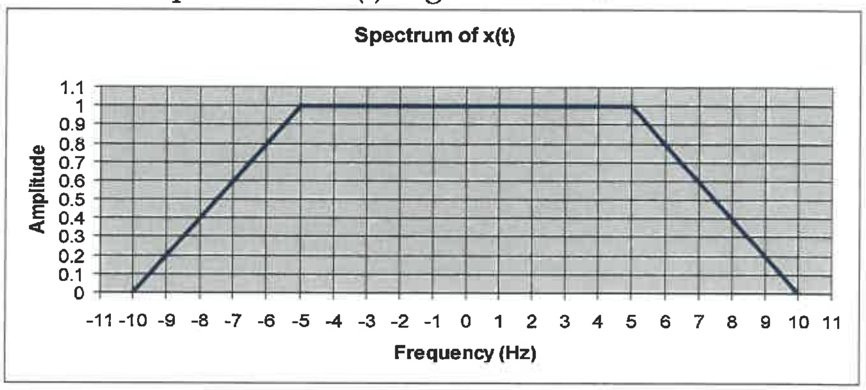
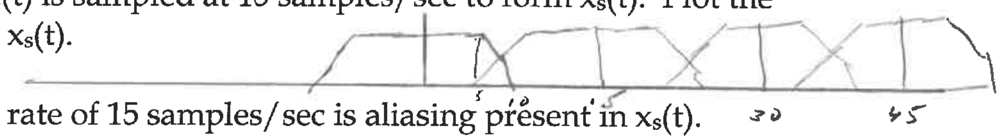
EECS 361
Final
Practice Problems

1. The spectrum of x(t) is given below:



a) The signal x(t) is sampled at 15 samples/sec to form $x_s(t)$. Plot the spectrum of $x_s(t)$.



b) At a sample rate of 15 samples/sec is aliasing present in $x_s(t)$. Circle **$\boxed{\text{YES}}$** or **NO** and justify your answer.

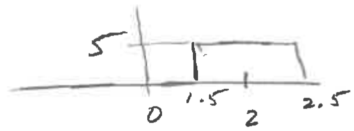
c) For x(t) given above, what is the minimum sample rate required to prevent aliasing? $f_s \geq 20$

d) If no aliasing is present, describe how x(t) is recovered from $x_s(t)$.

$x_s(t)$ — $\boxed{\text{ILPF} \; B = B_x}$ — $x(t)$

2.

a) Plot $x(t)=5\text{rect}(t-2)$.



b) Find the energy and power in $x(t)=5\text{rect}(t-2)$.

$E_x=$ 25 $P_x=$ 0

c) Is $x(t)=5\text{rect}(t-2)$ a power or energy signal? Circle **power signal** or **energy signal.**

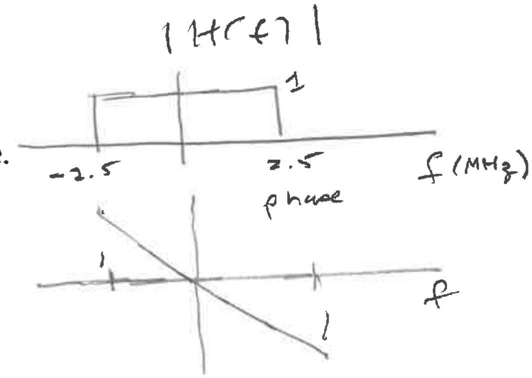
d) Is the signal $x(t)=5\text{rect}(t-2)$ periodic or aperiodic? Circle **periodic** or **aperiodic**

3. An ideal lowpass filter H(f) has a bandwidth B_H=2.5 MHz

The input to H(f) is x(t):

$$x(t)=\sum_{k=-\infty}^{\infty} rect\left(\frac{t-kT}{\tau}\right) \quad \text{where } \tau=0.25\mu s \text{ and } T=1\mu s$$

a) Sketch H(f), both the amplitude and phase response.



b) Sketch |X(f)|.

c) Find the system output y(t).

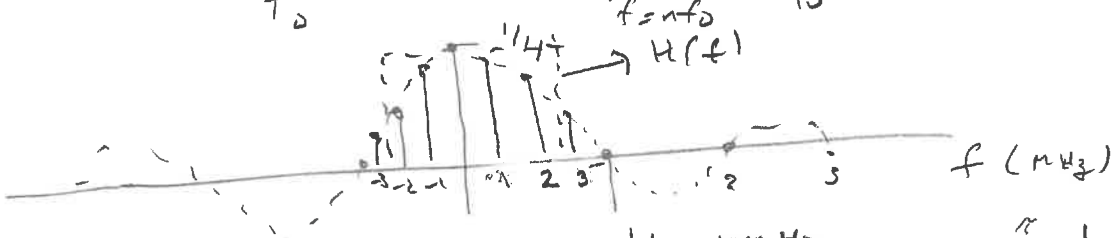
d) Find the power in the first harmonic of x(t).

$rect(t/\tau) \longleftrightarrow \tau\, sinc\left(\frac{\omega\tau}{2}\right) = \tau\, sinc\left(\frac{2\pi f\tau}{2}\right) = \tau\, sinc(\pi f\tau)$

$x(t) = \sum x_n e^{jn\omega_0 t} \qquad \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

$X_n = \frac{1}{T_0}\, \tau\, sinc(\pi f\tau)\Big|_{f=nf_0} = \frac{\tau}{T_0}\, sinc\left(\frac{n\pi\tau}{T_0}\right) = \frac{1}{4}\, sinc\left(\frac{n\pi}{4}\right)$

b)



$f_0 = 1MHz \qquad 1/\tau = 4MHz$

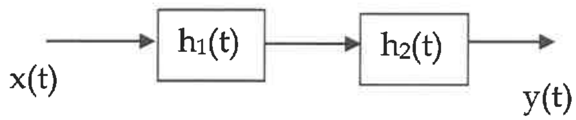
$X_0 = \frac{\tau}{T_0} = \frac{1}{4} \qquad X_1 = \frac{1}{4}\, sinc(\pi/4) = .225 \qquad X_2 = \frac{1}{4}\, sinc\left(\frac{\pi}{2}\right) = .16$

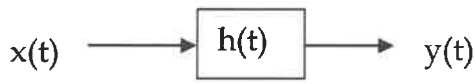
c) $y(t) = \sum_{n=-3}^{3} \frac{1}{4}\, sinc\left(\frac{n\pi}{4}\right) e^{j2\pi n f_0 t} = \frac{1}{4} + .45\cos(2\pi\, 1 f_0 t) + .32\cos(2\pi\, 2 f_0 t)$

d) $2|x_n|^2$

$2\left(\frac{1}{4}\, sinc(\pi/4)\right)^2 = 2\left(\frac{1}{4}\, sinc(\pi/4)\right)^2 = 2\left(\frac{1}{4}\, 0.9\right)^2 = 0.101$

4. A system is formed by the connecting two linear time-invariant subsystems, $h_1(t)$ and $h_2(t)$ in cascade.





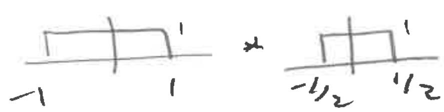
Where:

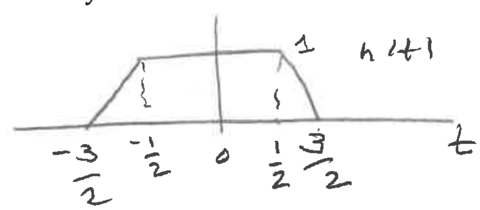
$$h_1(t) = rect\left(\frac{t}{2}\right) \quad \text{and} \quad h_2(t) = rect(t)$$

a) Find the impulse response for the overall system, h(t).

b) Find the transfer function, H(f), for the overall system.

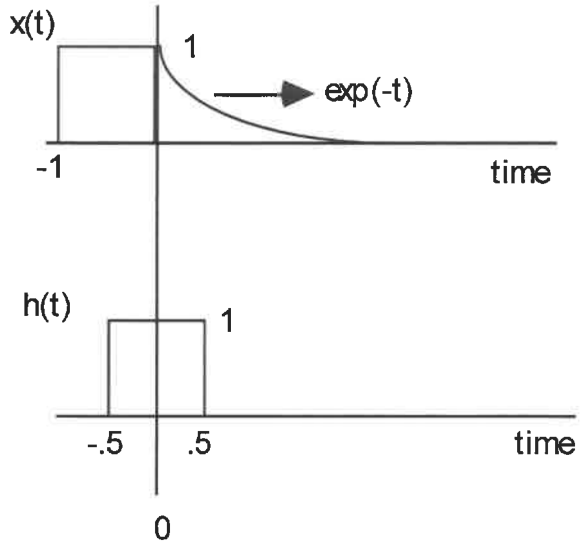
a) $rect(t/2) * rect(t) =$





b) $H(\omega) = \left(2\,\text{sinc}\left(\frac{\omega}{2}\right)\right)\left(\text{sinc}(\omega)\right)$

5. Consider a linear time invariant system with a impulse response of h(t), and input signal x(t) given below.



a) Give the range of values of time for which the output signal is non-zero.

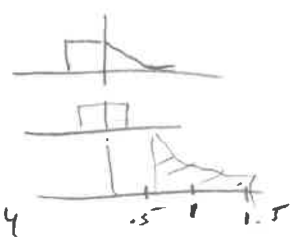
$-\frac{3}{2} < t < \infty$

b) Find y(1)

c) Find the Fourier Transform of x(t), i.e., X(f).

$y(1) \Rightarrow$

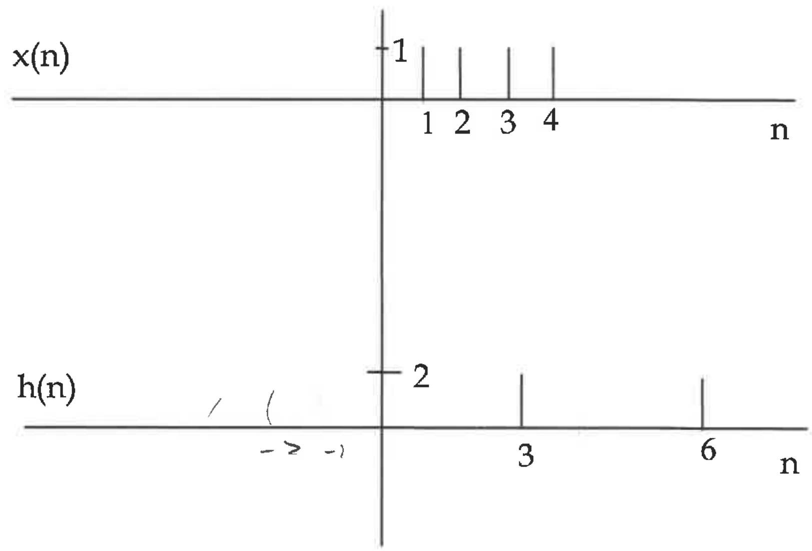
$\int_{.5}^{1.5} e^{-\tau} d\tau = 0.384$



c) $x(t) = \text{rect}(t + \frac{1}{2}) + u(t)\, e^{-t}$

$\longleftrightarrow \text{sinc}(\frac{\omega}{2})\, e^{+\frac{\omega}{2}} + \frac{1}{1+j\omega}$

6. The discrete time signal x(n) is input to a discrete time LTI system with impulse response h(n), x(n) and h(n) are zero where not specified below.



a) Find the output signal y(n) by performing the discrete time convolution of x(n) with h(n).

{0 0 0 0 2 2 2 4 2 2 2 3

b) Can you find y(n) using a 6-point DFT, that is, a DFT of x(n) and h(n) is calculated; these DFT's are multiplied and the IDFT of the product formed only using 6 values x(n) [0… 6] and h(n) [0… 6]? Circle **YES** or **NO** and justify.

7.

a) A music signal has a bandwidth of about 20 kHz. A DFT is use to analyze the frequency content of a music signal with a frequency resolution of 10 Hz. To achieve this frequency resolution what is the required record length in seconds.

$L = \frac{1}{10} = 100ms$

b) Given the record length, L (sec) found in part a) how many sample are in the record assuming the signal is sampled at 45 kHz. $(45 \times 10^3)(.1) = 4500$

c) Suppose a test tone of 3 kHz is used with a sample rate of 8000 samples/sec and the record length L=34.15ms, that is x(t) = cos(2π3000t) is sampled at 8000 samples/sec for L sec to form x(m).
Will the DFT of x(m)=X(n) contain a component only at 3 KHz, that is, will X(n) be zero except at 3000Hz. Circle **YES** or **NO** and justify your answer.

$T_0 = \frac{1}{3000}$ $\quad \frac{34.15 \times 10^{-3}}{(1/3000)} = 102.45$ not integer

$\Delta f = 29.2 \neq 30$

8. A system is given by the following input/output relationship:

$y(t) = x^3(t) + x^2(t) + 5$

Is the system:

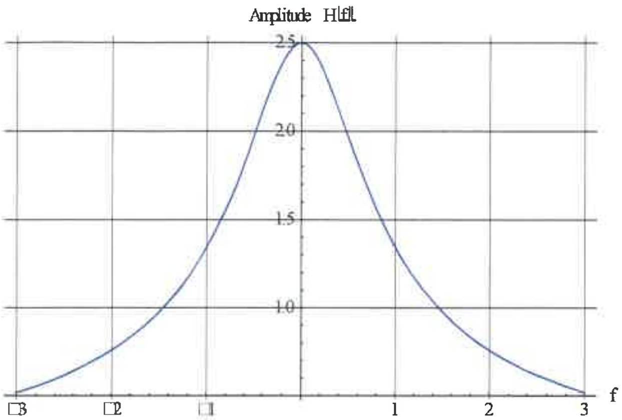
a) Linear Yes or No

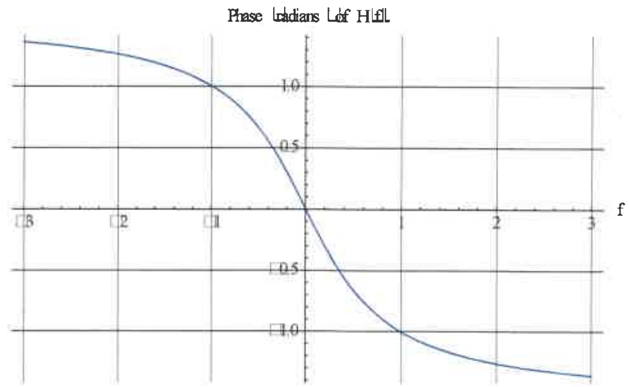
b) Time Invariant Yes or No

c) Memory-less Yes or No

9. A linear time invariant system has the following transfer function:

$$H(f)=\frac{10}{4+j2\pi f} \longleftrightarrow 10u(t)\,e^{-4t}$$





a) Find the impulse response for this system.

$h(t) = 10\,u(t)\,e^{-4t}$

b) Is this system casual? **Circle** $\boxed{\textbf{YES}}$ **or NO,** Justify your answer.

$h(t) = u(t)\,h(t)$

c) Is this system stable? **Circle** $\boxed{\textbf{YES}}$ **or NO,** Justify your answer.

$\int_0^{\infty} |h(t)|^2$ finite

d) Find the output signal, y(t), when the input signal is $x(t)=2\cos(2\pi t)+\cos(2\pi 2t)$. For partial credit give as much information about y(t) as possible.

$$H(f)\Big|_{f=1} = \frac{10}{4+j2\pi} = 1.3\,e^{-j1}$$

$$H(f)\Big|_{f=2} = \frac{10}{4+j4\pi} = .75\,e^{-j1.26}$$

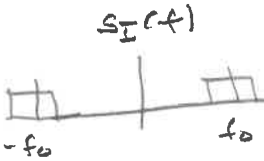
$$\mathbf{1.3 * 2 * \cos(2\,\pi t - 1) + 0.75\,\cos(2\,\pi 2t - 1.26)}$$

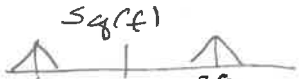
$$\mathbf{2.6 * \cos(2\,\pi t - 1) + 0.75\,\cos(2\,\pi 2t - 1.26)}$$

10. Let of $s_I(t) = x(t)\cos(2\pi f_o t)$ and $s_q(t) = z(t)\sin(2\pi f_o t)$, and $s(t) = s_I(t) + s_q(t) = x(t)\cos(2\pi f_o t) + z(t)\sin(2\pi f_o t)$, assume that both x(t) and z(t) have a 10KHz bandwidth and that f_o=1 MHz. Assume

$$X(f) = rect(\frac{f}{20000}) \text{ and } Z(f) = tri(\frac{f}{10000}).$$

a) Sketch the amplitude spectrum of $s_I(t) = x(t)\cos(2\pi f_o t)$ and $s_q(t) = z(t)\sin(2\pi f_o t)$.

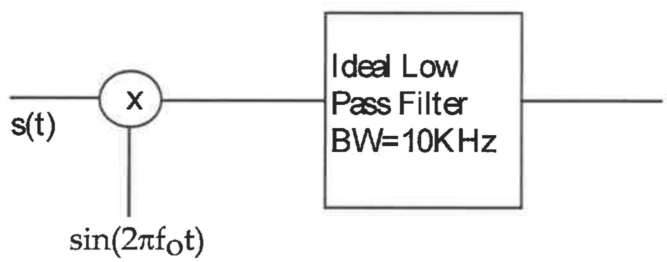




b) Sketch the amplitude spectrum of

$s(t) = s_I(t) + s_q(t) = x(t)\cos(2\pi f_o t) + z(t)\sin(2\pi f_o t)$.

c) Find the output y(t) of the following system in terms of x(t) and z(t):



$= \frac{z(t)}{2}$

d) Discuss the spectral occupancy of s(t), $s_I(t)$, and $s_q(t)$? What property of signals can be used to explain the result of part c) assuming that an ideal low pass filter is an approximation for an integrator?

- $S_I(t)$ and $S_q(t)$ share the same spectrum.

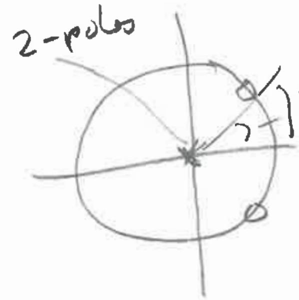
- orthogonality

$\int \cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt = 0$

11. Given x[n] = {0,2, 1, 2} for n=0, 1, 2, 3 and given a system transfer function of

$$H(z) = \frac{z^2 - 1.732z + 1}{z^2}$$

a. Find X(z) $= 2z^{-1} + z^{-2} + 2z^{-3}$



b. Find the poles and zeros of H(z) and draw the pole/zero diagram. 2 poles at $z = 0$

$\frac{1}{2}\left(-1.73 \pm \sqrt{(1.73)^2 - 4}\right) = \frac{-1.73}{2} \pm j.5$

$z^2 - 1.73z + 1 = (z + (0.86 - j0.5))(z + (0.86 + j0.5))$

$= (z + e^{j\frac{\pi}{6}})(z + e^{-j\frac{\pi}{6}})$

$\frac{\pi}{6} = 30°$

c. Is H(z) a stable system, justify?

Yes poles inside unit circle

d. Find the impulse response h[n] given H(z) given above.

e. Find the inverse z-transform of Y(z)=X(z)H(z). Describe a technique that can be used to check this answer (recommend you check your answer using this technique).

f. Find the system output y[n] for an input $x[n] = \cos(\frac{n\pi}{6})$

d) $H(z) = 1 - 1.73z^{-1} + z^{-2}$

$h[n] = \{\underline{1}\ \ -1.73\ \ 1\}$

e) $(2z^{-1} + z^{-2} + 2z^{-3})(1 - 1.73z^{-1} + z^{-2})$

y[n] = **{<u>0.0</u>, 2.0, −2.46, 2.27, −2.46, 2.0}**

f) $\omega_1 = \pi/6$

$y[n] = |H(e^{j\pi/6})| \cos\left(\frac{n\pi}{6} + \angle H(e^{j\pi/6})\right)$

$H(e^{j\pi/6}) = 1 - 1.73e^{-j\pi/6} + e^{-j\pi/3} \simeq 0$

12. Draw a line between the pole-zero diagram and its corresponding frequency response. O is the location of the zeros and X the locations of the poles.